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DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

by

E. H. Wedemeyer

August 1966

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BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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RDT&E Project No. 1P014501B11A

ABERDEEN PROVING GROUND, MARYLAND

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EHWedemeyer/so
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August 1966

DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

ABSTRACT

A theory is presented for the approximate computation of eigen-frequencies of liquid oscillations in non-cylindrical cavities. The eigen-frequencies are essential for the prediction of stability of liquid-filled shell. The theory reduces the problem of finding the eigen-value to a simple integration which can be performed by hand computation when the shape of the cavity is known. Comparison of theoretical prediction and available experimental data shows very good agreement.

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I. INTRODUCTION

The dynamic behavior of liquid-filled, spinning shell can be reliably predicted if the cavity occupied by the liquid is cylindrical or if the cavity is spheroidal and completely filled with liquid. The latter case, a spheroidal cavity completely filled with liquid, is of relatively little practical interest for the shell designer. In fact most cavities are nearly cylindrical and, furthermore, cavities are usually not 100 percent filled. It is, therefore, very fortunate that the case of a partially filled cylindrical cavity is accessible to theoretical analysis.

The theory for this case was published by Stewartson[1*] in 1959 and has since become the most important tool for the design of stable shell. Although most practical cavities are approximately cylindrical, some deviate sufficiently from exact cylindrical shapes that there is some doubt whether treating them as cylinders is still a good approximation. To clarify this point, Karpov[2] has made extensive experimental investigations of non-cylindrical cavities. (All cavities considered are bodies of revolution.) It was found that rounded corners produce very little effect on the range of instability but that considerable changes result from modifications like conical reduction of one or both ends of the cavity. In view of the large variety of cavity shapes, it is not possible to explore the effect of cavity shape solely on an experimental basis.

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* *Superscript numbers denote references which may be found on page 20.*

On the other hand, an exact theoretical approach to the problem appears to be quite hopeless for the following reason: The equations of the fluid motion are to be solved for boundary conditions that are imposed by the shape of the cavity walls. If the cavity is cylindrical, and only in this case, the solution is separable in radial and axial components. The tremendous simplification gained by the separation of variables is lost if the cavity shape deviates, however slightly, from a perfect cylinder. Solutions could be found, possibly, by numerical methods; however, such an approach is quite impractical considering that the computation cannot be performed prior to the cavity design. The best to be done within the borders of an exact theory is to compute the desired data, e.g., eigen-frequencies - for a comprehensive class of cavity shapes. The shell designer would then approximate the actual cavity by one for which data had been computed and tabulated. However, the advantage of an exact solution is lost when the cavity is only approximate, and one might ask whether an approximate solution for the exact cavity is not preferable.

II. THEORY OF APPROXIMATE EIGEN-FREQUENCIES

Of particular importance for the stability of the shell are the eigen-frequencies or frequencies of free oscillation of the liquid. According to Stewartson [1], instability occurs whenever any of the eigen-frequencies falls within a certain bandwidth about the frequency of nutation of the shell. Details on this stability theory are found in Stewartson's paper. [1] Whether a shell is stable can be predicted

when the eigen-frequencies and the residues of the forcing term, which determine the bandwidth, are known. For cylindrical cavities, the eigen-frequencies and residues are computed exactly and given in Stewartson's tables.[1,3] For small deviations from cylindrical shape, small changes of the eigen-frequencies and the residues must be expected. While accurate values for the eigen-frequencies are essential for the prediction of the dynamic behavior, the exact values for the residues are less important, since these determine only the bandwidth and usually the latter is affected by other factors, such as viscosity, more than by deviations from cylindrical shapes. From Karpov's experimental data[2] of un-damping rates, it appears that it is sufficient and safe to assume a value for the residue equal to that of an "equivalent" cylinder, i.e., one with the same volume and height. Thus, we are left with the determination of the eigen-frequencies for non-cylindrical cavities. The following analysis rests on the assumption that the radius of the cavity, a, is a slowly varying function of the distance along the axis, z, i.e.,

$$\left|\frac{da}{dz}\right| \ll 1 \qquad (1)$$

From Stewartson's analysis it follows that the oscillatory part of the pressure - which results from the liquid oscillations - is of the form:

$$p = P(r,z)\, e^{i(\omega t-\theta)} \qquad (2)$$

where P(r,z) satisfies the differential equation:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} - \frac{1}{r^2} P = \alpha^2 \frac{\partial^2 P}{\partial z^2} \tag{3}$$

with

$$\alpha^2 = \frac{4}{(1 - \tau)^2} - 1 \tag{4}$$

ω is the eigen-frequency, (r, θ, z) are polar coordinates and τ is the dimensionless eigen-frequency, $\tau = \frac{\omega}{\Omega}$, where Ω is the frequency of spin.

Let us use the notation (u, v, w) for the components of the oscillatory velocity in the polar coordinate system (r,θ,z). Then (u, v, w) can be expressed by certain linear combinations of the pressure and its partial derivatives $\frac{\partial p}{\partial r}$ and $\frac{\partial p}{\partial z}$.

We assume that the cavity shape is given by:

$$a = a(z) , \qquad 0 \le z \le 2c \tag{5}$$

and that Equation (1) holds for $0 \le z < 2c$. If the cavity is partially filled with liquid it is assumed, as in Stewartson's case, that the undisturbed free surface is a cylinder of radius b. For the following analysis we must assume that

$$b < a(z) \qquad \text{for} \qquad 0 \le z \le 2c \tag{6}$$

Equation (3) must be solved in connection with certain boundary conditions, e.g.,

$$u = \frac{da}{dz} w \qquad \text{at} \qquad r = a(z) \tag{7}$$

$$w = 0 \quad \text{at} \quad z = 0,\ 2c \tag{8}$$

and one boundary condition at $r = b$, which requires that the pressure be zero on the free surface when the cavity is partially filled. For a better understanding, a brief review of the eigen-value problem for Stewartson's case is given in the following: For the cylindrical cavity, i.e., if $\frac{da}{dz} = 0$, the solution of (3) with appropriate boundary conditions can be found by separation of variables:

$$P = C(\alpha kr) \ . \quad \cos kz \tag{9}$$

where $C = A \cdot J_1(\alpha kr) + B \cdot Y_1(\alpha kr)$ and J_1, Y_1 are Bessel functions of the first and second kind with the argument αkr. The solution (9) must satisfy certain boundary conditions at $r = b$ and $r = a$. Without giving a detailed derivation (which can be found in Reference 1) it suffices here to state the following results: For given b^2/a^2 and τ the boundary conditions at $r = b$ and $r = a$ lead to a transcendental equation for ka with discrete roots:

$$ka = \eta_n \ (b^2/a^2 \ , \tau) \ , \quad n = 1, 2, 3, \ldots . \tag{10}$$

where n is related to the number of radial waves of the solution (9).

The boundary conditions at $z = 0$, $2c$ lead to a similar equation for kc with roots:

$$kc = \frac{\pi}{2} [2j+1] \ j = 0, 1, 2 \ldots \tag{11}$$

Since w is proportional to $\frac{\partial p}{\partial z}$, Equation (8) can be written:

$$\frac{\partial p}{\partial z} = 0 \quad \text{at} \quad z = 0, 2c \quad . \tag{12}$$

Equations (12) and (9) yield:

$$\sin k\, 2c = 0 \ , \qquad kc = \frac{\pi}{2} \cdot m \tag{13}$$

Because of certain symmetry requirements, the number of half-waves in the z-direction must be odd, i.e., $m = (2j+1)$ and Equation (11) follows from (13). Comparison of (11) and (10) gives a condition for the fineness ratio:

$$\frac{c}{a[2j+1]} = \frac{\pi/2}{\eta_n(b^2/a^2, \tau)} \tag{14}$$

Equation (14) expresses that only certain discrete fineness ratios $\frac{c}{a}$ exist for given b^2/a^2 and τ or, for given b^2/a^2 and c/a there exists a set of discrete eigen-functions τ_{jn} according to the choice of j and n in Equation (14). In Stewartson's tables[1] the various $\frac{c}{a(2j+1)}$ are tabulated as functions of b^2/a^2 and τ. Let us call these functions $S_n(b^2/a^2, \tau)$, i.e.,

$$\frac{c}{a(2j+1)} = S_n(b^2/a^2, \tau) = \frac{\pi/2}{\eta_n(b^2/a^2, \tau)} \tag{15}$$

Returning to the eigen-value problem for the non-cylindrical cavity, we notice that the only difference is in the boundary condition (7). This condition states that the velocity component normal to the wall $r = a$ is zero. If now the inclination of the wall toward the axis is small, i.e., $\left|\frac{da}{dz}\right| \ll 1$, an obvious approximation of Equation (7) is:

$$u = 0 \qquad \text{at} \qquad r = a(z) \tag{16}$$

The form of the approximate boundary condition (16) suggests searching for approximate solutions of Equation (3) which are of the same general form as Equation (9) except that the radial wave number, k, should be a slowly varying function of z according to the fact that the radius, a, changes slowly with z. Thus, we try as an approximate solution of (3):

$$P = C(\alpha kr) \; . \; \cos \Phi(z) \tag{17}$$

where C is a cylinder function of the argument αkr and k depends weakly on z . The phase $\Phi(z)$ can no longer be assumed to be equal to kz but rather $\Phi = \int_0^z k d\zeta$.

It can be verified easily that (17) approximately satisfies the differential Equation (3) provided that:

$$k(z) = \frac{d\Phi}{dz} \tag{18}$$

and

$$\left|\frac{dk}{dz}\right| \ll k^2 \tag{19}$$

Since the boundary condition at $r = b$ and the approximate boundary condition at $r = a$ (Equation (16)) are - locally - the same as for the cylindrical cavity, we arrive naturally at the same condition for ka, viz., according to Equation (10),

$$k = \frac{1}{a} \eta_n \left(b^2/a^2, \tau\right) \tag{20}$$

Equation (20) gives the z- dependent wave number $k(z)$ and, at the same time, shows that the condition (19) is a consequence of $\left|\frac{da}{dz}\right| \ll 1$ (since η_n and its derivative with respect to b^2/a^2 are of order unity).

Thus, we conclude that (17) with (18) approximately satisfies the differential Equation (3) <u>and</u> the boundary conditions at $r = b$ and $r = a$.

Finally, the boundary condition (12): $\frac{\partial p}{\partial z} = 0$ at $z = 0, 2c$ requires that $\sin \Phi = 0$ at $z = 0, 2c$ or:

$$\Phi(0) = 0 \tag{21}$$

$$\Phi(2c) = \int_0^{2c} k(z)\, dz = \pi[2j+1] \tag{22}$$

Equation (22) can be considered as a generalization of Equation (11). Substituting (20) into (22) gives:

$$\pi[2j+1] = \int_0^{2c} \frac{1}{a} \eta_n \left(b^2/a^2, \tau\right) dz \tag{23}$$

Equation (23) connects b and τ, i.e., if the radius of the cylindrical void, b, is given, (23) determines τ and vice versa.

According to Equation (15) the function η_n is related to S_n, a function which is tabulated in Stewartson's tables. Thus, it is convenient to rewrite (23) by substituting η_n according to Equation (15) into (23) and, after dividing by $\pi(2j+1)$, one obtains:

$$1 = \frac{1}{2c} \int_0^{2c} \frac{c/a(2j+1)}{S_n\,(b^2/a^2\,,\ \tau)}\, dz \tag{24}$$

Equation (24) is easy to memorize: $S_n\,(b^2/a^2\,,\ \tau)$ is just the $c/a(2j+1)$-value of a cylinder with eigen-frequency τ and fill-ratio b^2/a^2. Thus, the integrand of (24) is the ratio of the local $c/a(2j+1)$ - value and the $c/a(2j+1)$ - value which would correspond to τ and the local b^2/a^2 - value. Equation (24) then states that the mean value of this ratio - averaged over the length of the cavity - should be equal to one.

Usually, the higher radial modes are unimportant, so that (24) must be solved only for $n = 1$. We will therefore write just S instead of S_n, having in mind that (24) is valid for any of the S_n values.

The evaluation of Equation (24) is difficult when S is given numerically (as in Stewartson's tables), since one has to assume both, b and τ, in order to perform the integration numerically and eventually repeat the procedure with changed values of τ (or b) until the correct value of τ (or b) can be observed by interpolation. For a 100 percent filled cavity, i.e., $b = 0$, the integration simplifies considerably, since S becomes independent of z and can be taken out of the integral. One obtains from (24):

$$S(0, \tau) = \frac{1}{2c} \int_0^{2c} \frac{c}{a(2j+1)} \, dz \tag{25}$$

Since S is the $\frac{c}{a(2j+1)}$ - value of a cylinder with eigen-frequency τ, Equation (25) can be interpreted in the following way: A completely filled non-cylindrical cavity has the same eigen-frequencies τ_{nj} as an "equivalent cylindrical cavity", which is defined by having a fineness ratio, c/a equal to the averaged c/a of the non-cylindrical cavity. For a 100 percent filled cavity, therefore, only one integration must be performed to determine the average c/a.

One way of solving the eigen-value problem given in Equation (24) is to approximate $1/S$ by a power series in b^2/a^2. If τ is given and b is to be determined one could plot $1/S$, for the particular τ given, versus b^2/a^2 and approximate the obtained curve by a polynomial in b^2/a^2. If neither τ nor b^2/a^2 are too large, the following formula is convenient and quite accurate:

$$\frac{1}{S} = \frac{1}{S_0} \left[1 + 1.26 \left(\frac{b^2}{a^2} \right)^2 \right] \tag{26}$$

S_0 is the value $S(0, \tau)$ as obtained from Stewartson's tables for $b^2/a^2 = 0$. The approximation (26) is valid within the following limits:

$$\begin{aligned} 0 &\le \tau \le 0.12 \\ 0 &\le b^2/a^2 \le 0.15 \end{aligned} \tag{27}$$

with (26) substituted into (24) one obtains:

$$S_o(\tau) = \frac{1}{2c} \int_0^{2c} \frac{c}{a(2j+1)} \left[1 + 1.26\left(\frac{b^2}{a^2}\right)^2\right] dz \qquad (28)$$

Equation (28) has the advantage that the right side is independent of τ and the left side independent of b, i.e., by integration of (28) one finds a relation of the form:

$$S_o(\tau) = C_1 + C_2\ b^4 \qquad (29)$$

A better approximation than (28) may be obtained by choosing a coefficient different from 1.26 in Equation (26) adapted to the particular τ, if τ is known. One could also include higher powers of b^2/a^2.

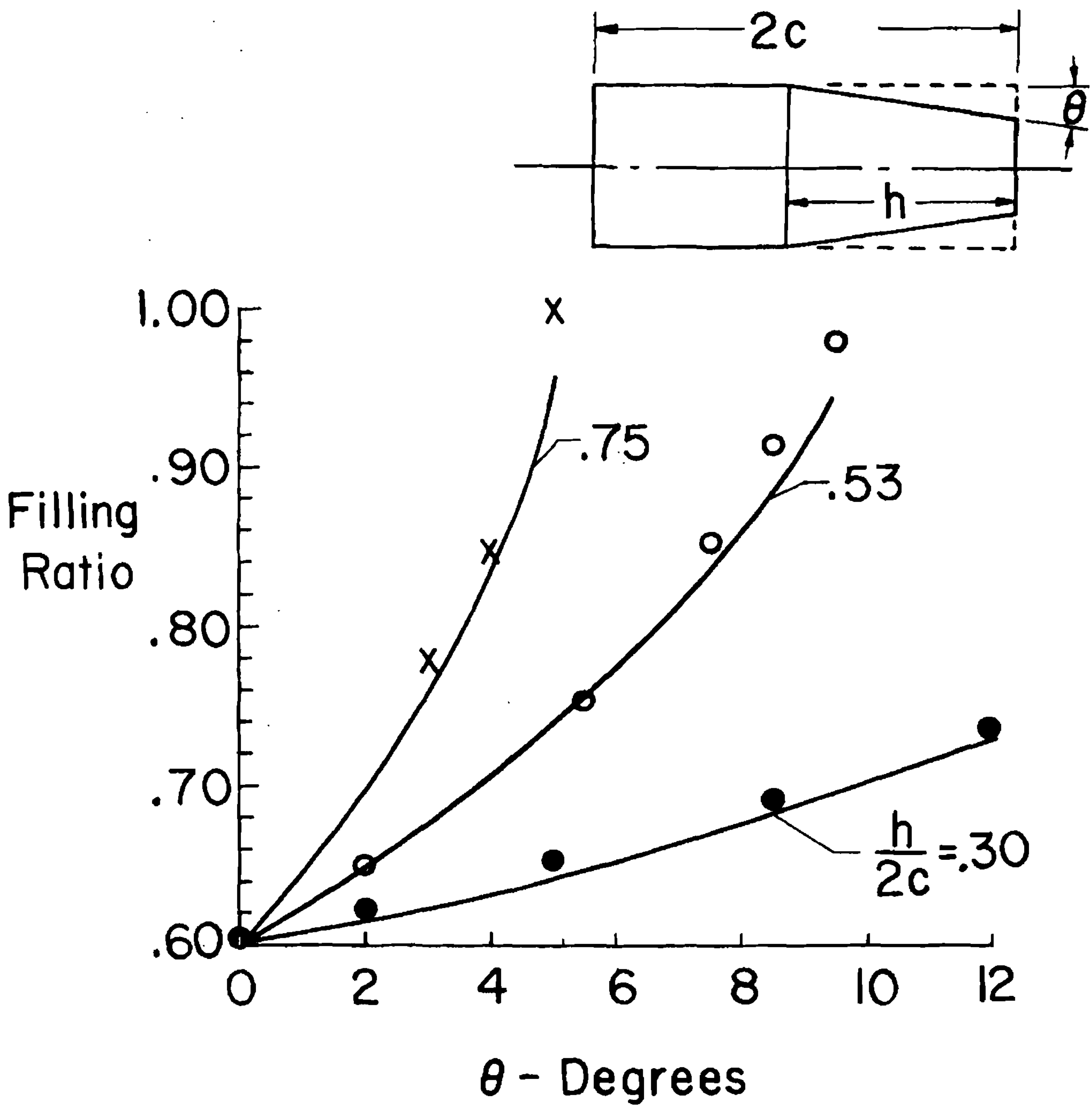
III. COMPARISON WITH EXPERIMENTS

In order to test the theory given in the preceeding section some of the experimental findings of Karpov[2] have been compared with theoretical predictions. The cavities investigated by Karpov were composed of a cylindrical and a conical section. A schematic of a cavity shape is shown in the following figure. In the diagram, the filling ratio which corresponds to a <u>fixed</u> eigen-frequency τ_0 is plotted versus the variable cone-angle θ. Three curves are shown that correspond to three different ratios of conical section h to overall length 2c. For $\theta = 0$ all cavities degenerate into a cylinder of fineness ratio $\frac{c}{a} = 2.687$ and the filling ratio $(1 - b^2/a^2)$ attains Stewartson's value. (It is $j = 1$, $n = 1$). The solid curves in the figure show the theoretically predicted values, the symbols the experimental data. The theory obviously gives good predictions even for θ-values for which the supposition on which the theory rests, viz. $|\frac{da}{dz}| = \ll 1$, is hardly fulfilled. The reason for this is, roughly, that the neglected terms in the boundary conditions and in the differential equation are orthogonal, or nearly so, to the other terms. Therefore, although the "local error" is of order $|\frac{da}{dz}|$, the "averaged error" over the length of the cavity can be very small. The theoretical curves in the figure were obtained by solving Equation (25) for fixed $\tau = 0.055$. To perform the integration, the following approximation was used:

$$\frac{1}{s} = 0.944 + 1.074 \left(\frac{b^2}{a^2}\right)^2$$

The integration could then be performed analytically and b^2/a^2 is found as a function of θ and $\frac{h}{2c}$.

Filling Ratio At Fixed Resonance Frequency $\tau_o = \tau_n = 0.055$



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